[Slide 1] Hello. This semester, I took an independent study course on Differential Geometry with Professor Yuan-Jen Chiang, during which I researched Local Surface Theory.

[Slide 2] Before we get into the basics, here are some surfaces. The upper left is something you'd see in Multivariable Calculus, and possibly the Helicoid at the bottom. The donut is a shape called a torus. We all look at the torus and appreciate its existence. The torus is a complicated creature made up of multiple coordinate patches (or simple surfaces).

[Slide 3] *Reads the first bullet point* $\mathcal{U}$ being an open subset of R squared means that for every point a b in $\mathcal{U}$, there's a number epsilon that's greater than zero such that the coordinates u one and u two – for the specifications of the coordinates of a surface – is in $\mathcal{U}$ whenever this equation *second bullet point* is true. So, a coordinate patch could be a circle. It's two-dimensional; it could be anything.

[Slide 4] *Reads all the bullets* The Kronecker symbol at the end is just zero if i and j aren't equal and one if they are.

[Slide 5] *Reads all the bullets* (Note: $\mathbf{x}_1$ and $\mathbf{x}_2$ are the notations for the partial derivatives of $\mathbf{x}$ with respect to u_1 and u_2, in that order.)

[Slide 6] A normal vector, magnitude not necessarily equal to one like the unit normal. When finding the equation of the tangent plane at a point, you arrange the values into the formula shown. The tangent space is the set of all tangent vectors to a surface and T sub P M is the representation of this, for future notice.

[Slide 7] The tangent plane of this simple surface was graphed in Mathematica at the point (1,2) using the methods shown previously.

[Slide 8] The first fundamental form of a surface is a matrix of metric coefficients at each point in the image of x. it's symmetric, meaning g sub i j equals g sub j i, which we'll see an example of next slide. Each coefficient is found by taking the inner product of the surface at a point with respect to the basis x one x two.

[Slide 9] Here, we see a simple surface I designated as capital V with coordinates u and w. The partial derivatives are displayed. Then the dot products are taken as such to get the coefficients of the matrix.

[Slide 10] The second fundamental form of a surface is a bilinear form on the tangent space given by this formula where X and Y are described like this and are both in the tangent space (because they're tangent vectors). The coefficients of the second fundamental form are the functions L i j defined on $\mathcal{U}$ by the inner product, or the dot product, of x i j and the unit normal vector. You may notice that this looks similar to the metric coefficients from the first fundamental form.

[Slide 11] Here, we see patch x being used to compute the coefficients of the second fundamental form. Just like with the metric coefficients, x one two and x two one, as well as L one two and L two one, the same, so only one calculation is shown for the pair.

[Slide 12] The Weingarten Map L is a function from the tangent space to R cubed that's equal to X times the normal vector when X is a tangent vector to the surface. You take the eigenvalues at a point to tell how the surface curves there.

[Slide 13] For a simple surface x with metric coefficients g i j, the Christoffel symbols are the functions of gamma as shown here for which g k l is the inverse of g i j. Now for an example of one.

[Slide 14] Here, we're continuing with the same example as a couple slides ago. Using the inner products of x i j and x k multiplied by the inverse matric coefficients summed over kk, we can obtain the Christoffel symbols of the patch. These can be further simplified, but this is how Mathematica displays them without extra work on my part.

[Slide 15] The image of a unit speed curve gamma lying on a surface has Frenet-Square apparatus kappa tau T N B. Capital N is not to be confused with the unit normal vector n. This is the normal vector field. The intrinsic normal of gamma is S is equal to the cross product of the unit normal and the tangent vector of gamma. This is important for understanding this relationship here. The curvature kappa with respect to s times the normal vector field is equal to the first derivative of the tangent of the unit speed curve is equal to the second derivative of the unit speed curve is equal to the normal curvature kappa n times the unit normal vector plus the geodesic curvature times the intrinsic normal from up here. We'll see the normal curvature and geodesic curvature later.

[Slide 16] If a simple surface x goes from $\mathscr{U}$ to R cubed, then the partial derivative of x with respect to u one times the partial derivative with respect to u two is equal to x i j, which is equal to this, and since for any unit speed curve, the normal curvature is equal to this, we can relate these together and get the final equation here.

[Slide 17] *Reads first three bullets* For a surface of revolution—which looks like…*draws a picture*—every meridian is a geodesic. *Starts drawing meridians* This is a geodesic. This is a geodesic. This is a geodesic. A circle of latitude is geodesic if and only if at all points tangent x one to the meridians is parallel to the axis of revolution. So, pretty much wherever the equators are. *Starts drawing latitudes* This is a very uniform surface of revolution. *That is sarcasm. It is, in fact, not even within throwing distance of such a description* All these tangent vectors are parallel to this axis of revolution. *Draws the axis of revolution*

[Slide 18] The normal curvature of a unit speed curve gamma with tangent T is the second fundamental form of T with itself. We draw back to the relationships described in the unit speed curve slide to describe kappa n in more simple terms. *Reads final two bullets*

[Slide 19] Back to the Weingarten map L, its eigenvalues at a point P are the principal curvatures kappa one and kappa two. *Reads the five remaining bullet points* Kappa one and kappa two make up the trace, because they are the eigenvalues.

[Slide 20] *Reads the slide, referring to $\tilde{X}$ as “interesting X”. Not the technical term.*

The twentieth slide seems to have been merged with my cited slide. I noticed this too late.